JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 21, No. 4, July–August 1998

Orbit Determination and Photogrammetry Using Mars-94 Tracking and Images

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A consistent approach for a combined evaluation of radio tracking data and three-line scanning stereo camera images is presented for use within the Mars-94 mission. Within this project, orbit determination and photogrammetric analysis are carried out with distinct software packages by independent working groups. The proposed approach ensures the proper utilization of orbit determination-based trajectory information in the photogrammetric bundle adjustment. Vice versa, it allows the incorporation of photogrammetric image information to improve the orbit determination and support the estimation of Mars science parameters. Focus is given to the combined approach, its mathematical description, and its main advantages.

I. Introduction

A. Mars-94 Stereo Camera Experiments

THE scientific payload of the Mars-94 mission comprises two German cameras, the high-resolution stereo camera (HRSC) and the wide angle optoelectronic stereo scanner (WAOSS). Both cameras are based on a three-line charge-coupled device (CCD) concept, which allows simultaneous scanning of the Martian surface in the forward-, nadir-, and backward-looking directions and provides a novel opportunity for direct stereo imaging of the Martian surface. For HRSC[1] a ground pixel resolution of 12 m is achieved, whereas WAOSS[2] provides a 96-m resolution with a swath width of 520 km, about eight times that of HRSC. Both cameras are mounted on the high-precision steerable Argus platform (see Fig. 1) for nadir-looking imaging over a wide range in true anomaly.

B. Photogrammetric Processing Concept

The in-depth photogrammetric processing of the HRSC and WAOSS images involves the determination of three-dimensional coordinates of ground points on the surface of Mars based on observed pixel coordinates for a multitude of conjugate points that are visible in at least two different image lines. Furthermore, the photogrammetric processing of HRSC/WAOSS data provides independent information on the position and attitude of the camera, based on the relation of image, object, and camera coordinates and the stereoscopic information content of three-line CCD images (see Fig. 2). To this end, the so-called bundle adjustment algorithm allows for the estimation of the camera's exterior orientation, i.e., its position and attitude, at selected points along the spacecraft trajectory.

For the sake of simplicity and operational convenience, orbit determination and bundle adjustment have traditionally been considered as distinct processes. From a scientific point of view, however, this separation suffers from several major drawbacks: 1) Considering the camera's exterior orientation at selected orientation images as independent estimation parameters results in a destabilization of the estimation process, if the spacing is close enough to allow smooth interpolation. 2) The physical relation between the exterior orientation at different orientation images is not accounted for. 3) Statistical properties of the navigation data resulting from the orbit determination are treated inconsistently. 4) Exterior orientation parameters estimated in the bundle adjustment do not allow an interpretation in terms of physical parameters. Therefore, to properly utilize the navigational information contained in the radio tracking data and the image data, both data types have to be evaluated in a combined estimation process.[3]

Because of the complexity of both the orbit determination process and the bundle adjustment, a direct combination of the corresponding software packages does not, however, appear to be feasible or adequate for the Mars-94 project. Instead, the trajectory data and the orbit determination parameter covariance matrix are incorporated into the photogrammetric processing, together with associated partial derivatives of the spacecraft position with respect to the epoch state and model parameters. The bundle adjustment normal equations, thus, may be formulated in terms of a physical trajectory model, which reduces the number of estimation parameters and allows a scientific interpretation of the navigational information contained in the image data. At the same time, the information content of the radio tracking data is properly accounted for by including the orbit determination covariance matrix as a priori information on the estimation parameters.

II. Orbit Determination

Orbit determination of the Mars-94 spacecraft basically covers the adjustment of a priori trajectory information using radiometric or other tracking data. It may be considered as a three-stage process involving the trajectory and trajectory partials computation, the measurement and measurement partials computation, and, finally, the least-squares correction of the initial state vector and free model parameters.

The trajectory and trajectory partials computation inside the orbit determination software involves the numerical integration of the six-dimensional, first-order initial value problem

$$\dot{y} = f(t, y, p), \qquad y_0 = y(t_0) \tag{1}$$

for the satellite's space-fixed state vector y at time t. Here the vector p is used to denote free parameters of the force model, e.g., the atmospheric drag coefficient, that shall be estimated within the orbit determination process.

The trajectory model is complemented by an appropriate measurement model, which relates the predicted value of the ith measurement to the corresponding state vector $y(t_i)$ at time t_i and a vector of adjustable measurement model parameters q, such as measurement or timing biases. Because $y(t_i)$ itself depends on the epoch state and the trajectory model parameters, the measurement model may further be expressed as

$$f_{zi} = f_{zi}(y_0, p, q) \tag{2}$$

Presented as Paper 94-3764 at the AIAA/AAS Astrodynamics Conference, Scottsdale, AZ, Aug. 1–3, 1994; received Nov. 2, 1994; revision received Aug. 14, 1996; accepted for publication March 25, 1998.

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which relates the predicted value f_{zi} of the ith measurement z_i to the epoch state vector y_0 and the vectors p and q of force and measurement model parameters.

The orbit determination problem may now be formulated as the nonlinear least-squares problem of minimizing the quadratic weighted norm

$$J_z = v_z^T W_{z,z} v_z \tag{3}$$

of the residuals, i.e., of the difference

$$v_z = f_z(y_0, p, q) - z \tag{4}$$

between the vector f_z of modeled measurements and the vector z of observed measurements. The matrix $W_{z,z}$ denotes a weighting matrix that is equal to the inverse covariance of the measurement vector z.

Expanding f_z around reference (initial) values y_0^0, p^0, and q^0 yields the linearized observation equation

$$v_z = A_{z,y}\Delta y_0 + A_{z,p}\Delta p + A_{z,q}\Delta q - l_z \tag{5}$$

for the required corrections

$$\begin{pmatrix} \Delta y_0 \\ \Delta p \\ \Delta q \end{pmatrix} = \begin{pmatrix} y_0 \\ p \\ q \end{pmatrix} - \begin{pmatrix} y_0^0 \\ p^0 \\ q^0 \end{pmatrix} \tag{6}$$

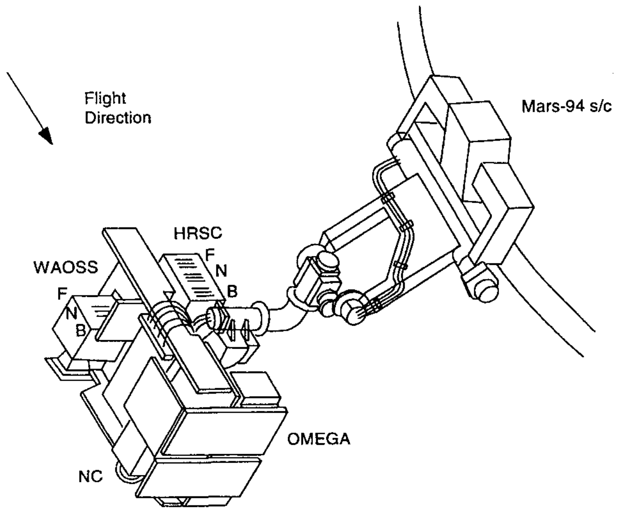

Fig. 1 HRSC and WAOSS experiments onboard the steerable Argus platform.

to the reference values of the estimation parameters. Here

$$l_z = z - f_z\left(y_0^0, p^0, q^0\right) \tag{7}$$

denotes the difference between observed and the modeled reference measurements, whereas the Jacobian

$$A_{z,ypq} = (A_{z,y}, A_{z,p}, A_{z,q}) = \left.\frac{\partial f_z(y_0, p, q)}{\partial(y_0, p, q)}\right|_{(y_0^0, p^0, q^0)} \tag{8}$$

denotes the partial derivatives of f_z with respect to the unknown parameters y_0, p, and q.

Substitution of the linearized observation equation (5) into the loss function (3) then yields the formal solution

$$\begin{pmatrix} y_0 \\ p \\ q \end{pmatrix} = \begin{pmatrix} y_0^0 \\ p^0 \\ q^0 \end{pmatrix} + \left(A_{z,ypq}^T W_{z,z} A_{z,ypq}\right)^{-1} A_{z,ypq}^T W_{z,z} l_z \tag{9}$$

of the least-squares adjustment problem. The corresponding (unscaled) covariance matrix

$$C_{ypq,ypq} = \left(A_{z,ypq}^T W_{z,z} A_{z,ypq}\right)^{-1} \tag{10}$$

is equal to the inverse of the information matrix, and the square roots of the diagonal elements of $C_{ypq,ypq}$ give the standard deviations of the estimated parameters y_0, p, and q.

To circumvent the effect of nonlinearities, the least-squares solution is iteratively refined with updated Jacobians by replacing the reference values with the resulting solution.

III. Photogrammetric Processing

Within the present section, the fundamental algorithms developed for the photogrammetric processing of images from Earth-orbiting three-line scanning stereo cameras such as the modular optoelectronic multispectral scanner-02 (MOMS-02) and the monocular electro-optical stereo scanner (MEOSS)[4,5] are described. After discussing the potential drawbacks of the conventional approach for evaluation of HRSC and WAOSS data that may arise from a weak surface texture, a low-precision ground control network, long-arc orbit paths, and the lack of an inertial navigation system in the context of the Mars-94 mission, the incorporation of orbital constraints and the combination of orbit determination and photogrammetry are presented in Sec. IV.

A. Image Coordinates and Exterior Orientation Modeling

As shown in Fig. 3, the first step in the photogrammetric restitution of HRSC and WAOSS data consists of finding a suitable set of conjugate points in the image strips resulting from the forward-, nadir-, and backward-looking sensor lines (see Fig. 4). Conjugate points, i.e., points that are visible in at least two image strips,

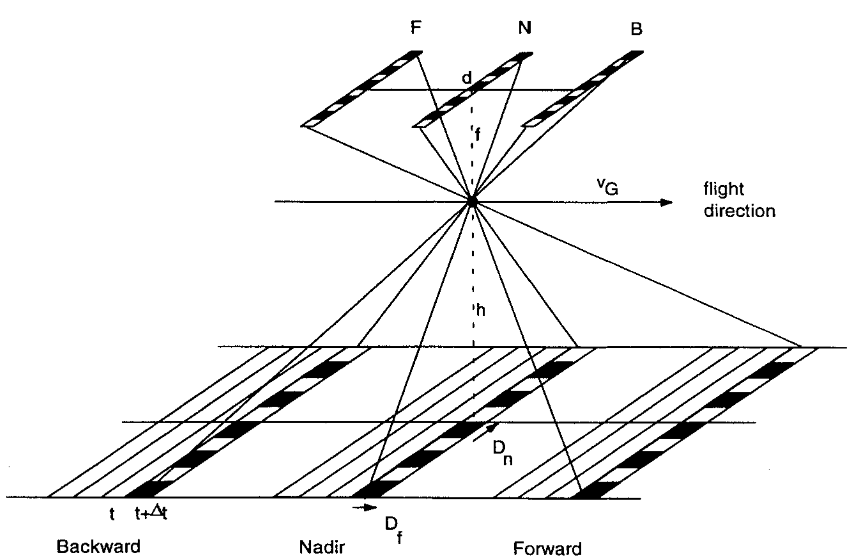

Fig. 2 Pushbroom operation of three-line CCD stereo scanners.

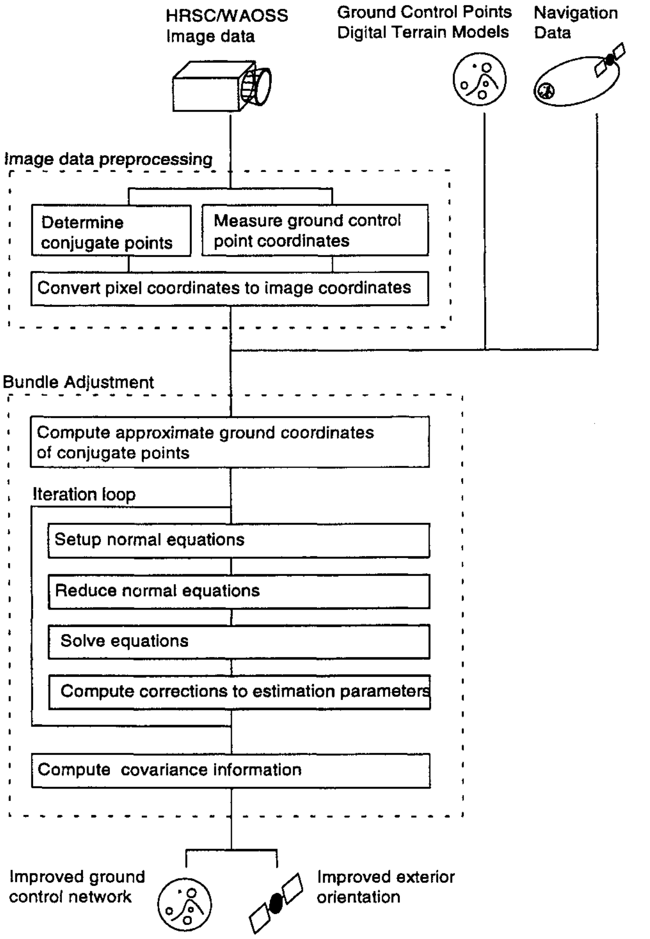

Fig. 3 Photogrammetric restitution scheme.

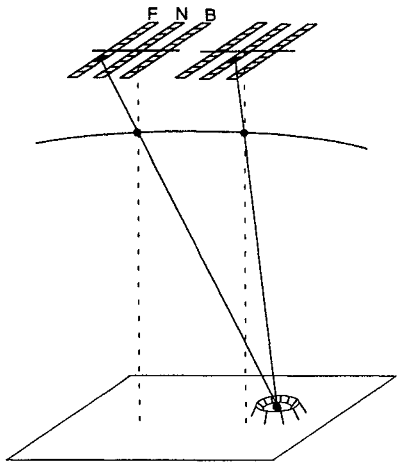

Fig. 4 Conjugate points represent common features in images strips from different CCD lines.

establish a link between images taken at different times and camera locations and, therefore, provide the basic information for stereoscopic processing. A large number of conjugate points is required for the rectification of a complete image sequence because each conjugate point is present in at most three images due to the three-line stereo camera concept.

The measured image coordinates (or pixel numbers) $\boldsymbol{u} = (u_x, u_y)^T$ of the conjugate points and the ground control points in the image plane of the camera constitute the basic measurements of the bundle adjustment process. Based on the collinearity equations

$$\boldsymbol{u} = \boldsymbol{u}^p - \frac{f}{\boldsymbol{d}_z^T \cdot (\boldsymbol{x} - \boldsymbol{x}^c)} \begin{pmatrix} \boldsymbol{d}_x^T \cdot (\boldsymbol{x} - \boldsymbol{x}^c) \\ \boldsymbol{d}_y^T \cdot (\boldsymbol{x} - \boldsymbol{x}^c) \end{pmatrix} \tag{11}$$

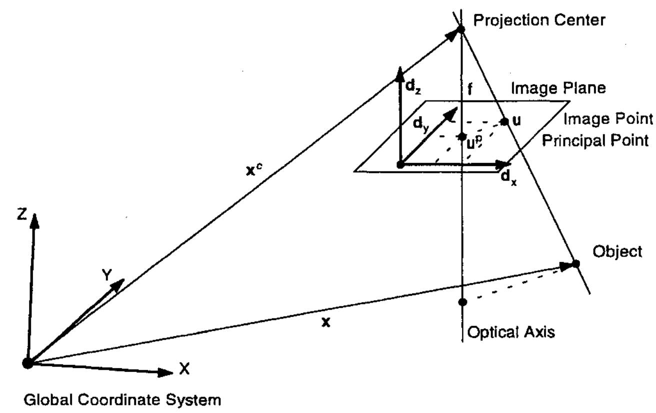

Fig. 5 Derivation of the collinearity equations.

(cf. Fig. 5), which relate the image coordinates to the object point coordinates $\boldsymbol{x}$ and the exterior orientation of the camera, the image coordinates may be modeled by a function

$$\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{x}^c(t), \boldsymbol{\theta}(t)) \tag{12}$$

of the object coordinates $\boldsymbol{x}$, the camera position $\boldsymbol{x}^c$, and the attitude $\boldsymbol{\theta}$ at the time of the image. The difference between observed and modeled image coordinates may then be used to improve both the object coordinates and the exterior orientation in a least-squares correction process.

Because it is not possible to estimate six exterior orientation parameters for each three-line image, so-called orientation points (or orientation images) are introduced. Based on an interpolation of neighboring orientation points, the camera's position

$$\boldsymbol{x}^c(t) = \boldsymbol{x}^c(t, \boldsymbol{X}^c) \tag{13}$$

and attitude

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t, \boldsymbol{\Theta}) \tag{14}$$

at arbitrary times t and the image coordinates

$$\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{x}^c(t), \boldsymbol{\theta}(t)) = \boldsymbol{u}(t, \boldsymbol{x}, \boldsymbol{X}^c, \boldsymbol{\Theta}) \tag{15}$$

of an object point $\boldsymbol{x}$ can be modeled in terms of the exterior position vector

$$\boldsymbol{X}^c = \left(\boldsymbol{x}_1^c, \boldsymbol{x}_2^c, \ldots, \boldsymbol{x}_n^c\right) \tag{16}$$

which comprises the camera positions $\boldsymbol{x}_i^c$ $(i = 1, \ldots, n)$ at selected orientation points and the corresponding attitude vector

$$\boldsymbol{\Theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_n) \tag{17}$$

at the orientation points.

Because the exterior orientation parameters $\boldsymbol{X}^c$ and $\boldsymbol{\Theta}$ are similarly handled within a formal solution of the photogrammetric normal equations, a single parameter $\boldsymbol{O} = (\boldsymbol{X}^{cT}, \boldsymbol{\Theta}^T)^T$ may be introduced instead. Denoting $\boldsymbol{X} = (\boldsymbol{x}_1^T, \ldots, \boldsymbol{x}_{n_X}^T)^T$ the vector of object coordinates and $\boldsymbol{O}$ the vector of the orientation parameters considered in the estimation, the modeled image coordinates $\boldsymbol{f}_U$ depend only on $\boldsymbol{X}$ and $\boldsymbol{O}$. The residuals can, therefore, be written as

$$\boldsymbol{v}_U = \boldsymbol{f}_U(\boldsymbol{X}, \boldsymbol{O}) - \boldsymbol{U} \tag{18}$$

where $\boldsymbol{U} = (\boldsymbol{u}_1^T, \ldots, \boldsymbol{u}_{n_U}^T)^T$ is the vector of image coordinates. Expanding $\boldsymbol{f}_U$ around the reference values $\boldsymbol{X}^0$ and $\boldsymbol{O}^0$ yields the linearized equation

$$\boldsymbol{v}_U = \boldsymbol{A}_{U,X} \Delta\boldsymbol{X} + \boldsymbol{A}_{U,O} \Delta\boldsymbol{O} - \boldsymbol{l}_U \tag{19}$$

for the residuals in terms of corrections $\Delta\boldsymbol{X} = \boldsymbol{X} - \boldsymbol{X}^0$ and $\Delta\boldsymbol{O} = \boldsymbol{O} - \boldsymbol{O}^0$ to the reference values. Here

$$\boldsymbol{l}_U = \boldsymbol{U} - \boldsymbol{f}_U(\boldsymbol{X}^0, \boldsymbol{O}^0) \tag{20}$$

denotes the difference between observed and the modeled image coordinate measurements, whereas the Jacobians

$$A_{U,X} = \frac{\partial f_U}{\partial X}, \qquad A_{U,O} = \frac{\partial f_U}{\partial O} \tag{21}$$

denote the partial derivatives of f_U with respect to the independent parameters X and O.

B. Normal Equations and Elimination of Object Coordinates

The least-squares bundle adjustment problem is equivalent to minimizing the weighted norm of the residuals v_U:

$$J_U = v_U^T W_{U,U} v_U \tag{22}$$

where $W_{U,U}$ denotes a weighting matrix that is equal to the inverse variance–covariance matrix of the measurement vector U.

To improve the condition of the bundle adjustment normal equations, a priori information on the exterior orientation may be incorporated into the least-squares estimation analogous to the use of additional observations. To this end a weighting matrix

$$W_{O,O}^a = \left(C_{O,O}^a\right)^{-1} \tag{23}$$

which is equal to the inverse of the a priori parameter covariance matrix, and the residuals

$$v_O = \Delta O - l_O \tag{24}$$

of the estimation parameters with respect to the a priori values are introduced. Here

$$l_O = O^a - O^0 \tag{25}$$

is the difference between the a priori parameters and the reference values employed in the linearization of the measurement equations.

Minimizing the extended loss function

$$J = v_U^T W_{U,U} v_U + v_O^T W_{O,O}^a v_O \tag{26}$$

then yields the normal equations

$$\begin{pmatrix} N_{X,X} & N_{X,O} \\ N_{O,X} & N_{O,O} \end{pmatrix} \begin{pmatrix} \Delta X \\ \Delta O \end{pmatrix} = \begin{pmatrix} A_{U,X}^T W_{U,U} l_U \\ A_{U,O}^T W_{U,U} l_U + W_{O,O}^a l_O \end{pmatrix} \tag{27}$$

with

$$N_{X,X} = A_{U,X}^T W_{U,U} A_{U,X}, \qquad N_{X,O} = N_{O,X}^T = A_{U,X}^T W_{U,U} A_{U,O}$$

$$N_{O,O} = A_{U,O}^T W_{U,U} A_{U,O} + W_{O,O}^a$$

for the unknown corrections ΔX and ΔO.

For a typical image sequence, a total of up to 100,000 conjugate points and 1000 orientation parameters may be required, which gives rise to a huge dimension of the normal equations. Making use of the specific structure of the normal equations, the least-squares problem may, however, be reduced by eliminating the conjugate points' object coordinates. Solving the first block of the normal equations for ΔX and substituting the resultant expression

$$\Delta X = (N_{X,X})^{-1}\left(A_{U,X}^T W_{U,U} l_U - N_{X,O}\Delta O\right) \tag{28}$$

back into the second block of equations, the orientation parameters are given by

$$\Delta O = \left(N_{O,O} - N_{O,X} N_{X,X}^{-1} N_{X,O}\right)^{-1}\left(A_{U,O}^T W_{U,U} l_U + W_{O,O}^a l_O - N_{O,X} N_{X,X}^{-1} A_{U,X}^T W_{U,U} l_U\right) \tag{29}$$

To evaluate this expression, the inverse of $N_{X,X}$ is required, which may, however, be obtained with moderate effort due to the block diagonal structure of the matrix. In the case of the photogrammetric evaluation of a single three-line CCD image session, each feature associated with a conjugate point is, in general, observed in three images. Therefore, 3×2 image plane coordinates have been recorded for a single conjugate point. Because the ground coordinates of a conjugate point consist of a three-element vector, the $N_{X,X}$ matrix is composed of 3×3 submatrices along the principal diagonal with all other elements outside these submatrices being zero. Thus, the matrix may be inverted with little effort, provided that the images are properly grouped within the vector U.

IV. Combined Orbit Determination and Bundle Adjustment

As has been pointed out in Refs. 6 and 7, an essential difference between satellite photogrammetry and conventional aircraft photogrammetry stems from the specific properties of satellite and aircraft trajectories. Whereas the camera position might, in principle, be randomly distributed in space on an aircraft flight path, it is known to follow the physical laws of motion on a satellite trajectory. Orbital constraints may, therefore, be incorporated into the photogrammetric bundle adjustment by requiring that the estimated camera position parameters for each image sequence satisfy the equation of motion with a priori initial conditions and covariances.

Given the equation of motion, the orbit of a spacecraft is completely determined by six orbital elements or the six-dimensional initial state vector at a specific epoch. Hence, the number of estimation parameters in satellite photogrammetry can be reduced considerably by replacing the ephemeris of exterior positions with the corresponding orbital parameters. As a result, one benefits from an improved point determination stability and accuracy and a decreased dependence on supplementary ground control information. At the same time, a mapping of image noise into the estimated spacecraft trajectory is avoided, which might otherwise occur due to the large number of estimated position parameters (Fig. 6).

So far, the use of orbital constraints in satellite photogrammetry, which has been formulated as early as 1960 by Brown (see Refs. 7 and 8) has mainly been utilized in the construction of a lunar control network from photographs obtained by a metric camera during the Apollo 15–17 missions.[9,10] For Earth-orbiting satellites, the use of short-arc orbital constraints has been studied by Salamonowicz[11] and Westin[12] for the rectification of Landsat and SPOT imagery.

Based on these considerations a concept for combined orbit determination and photogrammetric restitution of HRSC/WAOSS images has been developed for use within the Mars-94 project, which is outlined in Fig. 7. Besides incorporating orbital constraints into the conventional bundle adjustment algorithm for three-line stereo sensor images, it is designed to support the estimation of Mars science parameters by exploiting the information content of radiometric tracking data and optical measurements in a common least-squares parameter estimation.

Inasmuch as a direct combination of orbit determination and bundle adjustment does not appear to be feasible or adequate for the Mars-94 project, due to the complexity of both processes, a simplified but statistically equivalent approach has been chosen. To this end trajectory information resulting from the orbit determination process is provided in a suitable form for subsequent use in the bundle adjustment. Aside from a continuous spacecraft ephemeris, this information comprises trajectory partials with respect to the initial state vector and trajectory model parameters. Preliminary parameter estimates obtained from radio tracking data within the conventional orbit determination, thus, may be improved within the subsequent bundle adjustment based on HRSC and WAOSS images. Additionally, the covariance information derived from the orbit determination process will be introduced in the bundle adjustment as a priori weight information.

A. Modeling of Image Coordinates in Terms of Physical Trajectory Parameters

For the inclusion of orbital constraints, the camera position

$$x^c(t) = x^c(t, y_0) \tag{30}$$

and the image coordinates

$$u = u(x, x^c(t)) = u(t, x, y_0) \tag{31}$$

have to be expressed as a function of the satellite's epoch state vector y_0, making use of the known laws of orbital motion. The estimation

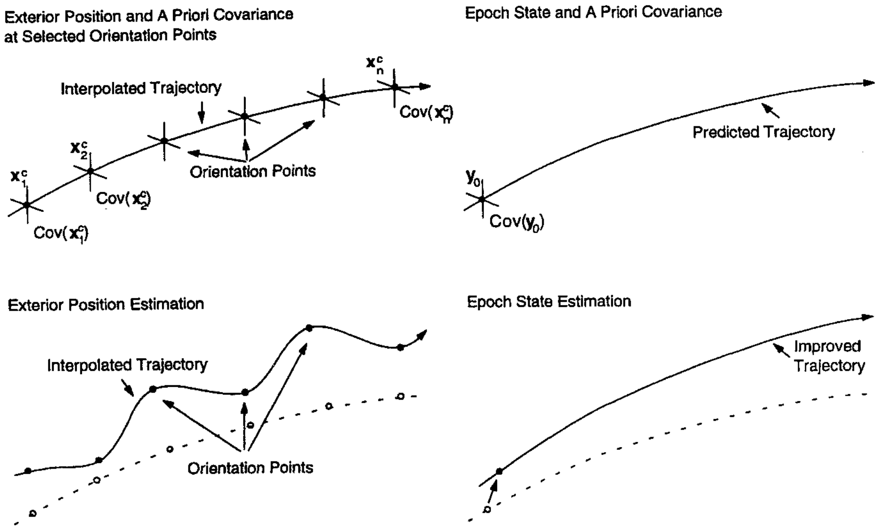

Fig. 6 Application of orbital constraints in the photogrammetric restitution of three-line stereo camera images: Although the conventional estimation and interpolation of selected exterior positions may result in an unrealistic trajectory (left), all camera positions are constrained to lie on a physical trajectory when estimating the satellite's epoch state vector (right).

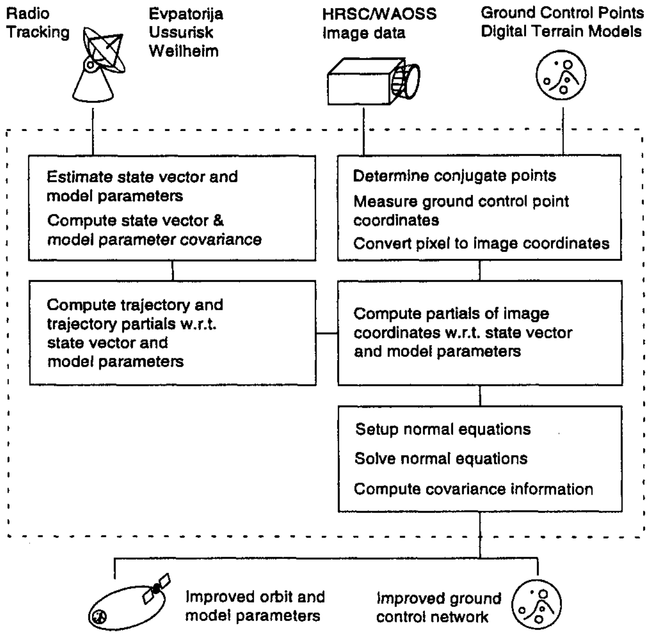

Fig. 7 Combined orbit determination and photogrammetry of HRSC/WAOSS images.

of the exterior position vector can then be replaced by an estimation of y_0, where the a priori value y_0^a and the associated covariance $C_{y,y}^a$ are provided as a result of the orbit determination derived from radio tracking data. Besides reducing the number of unknowns in the bundle adjustment, the estimated camera position parameters are, thus, constrained to satisfy the equation of motion during the whole imaging sequence (Fig. 6).

For the photogrammetric restitution of HRSC and WAOSS images, the basic concept of orbital constraints is, furthermore, extended by incorporating additional model parameters, which may then be estimated in the bundle adjustment. The set of additional parameters comprises both force model parameters $\boldsymbol{p}$ involved in the modeling of the inertial spacecraft trajectory and planetary rotation parameters ω that affect the transformation from inertial to body-fixed positions. To this end the camera position with respect to the Mars-centered, Mars-fixed coordinate system is expressed as

$$\boldsymbol{x}^c(t) = \boldsymbol{x}^c(t, \boldsymbol{y}_0, \boldsymbol{p}, \omega) = \boldsymbol{\Omega}(t, \omega)\boldsymbol{r}(t, \boldsymbol{y}_0, \boldsymbol{p}) \tag{32}$$

where $\boldsymbol{r}$ is the inertial position vector of the camera with respect to the Earth mean equator and equinox of J2000 and $\boldsymbol{\Omega}$ denotes the rotation matrix describing the transformation into the body-fixed coordinate system.

Because of the lack of a dynamical model describing the camera's attitude $\boldsymbol{\theta}(t)$, the concept of exterior orientation points is maintained when combining orbit determination and bundle adjustment in the way described here. The resulting observation equation for the image coordinates is then given by

$$\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{x}^c(t), \boldsymbol{\theta}, \boldsymbol{c}) = \boldsymbol{u}(t, \boldsymbol{x}, \boldsymbol{y}_0, \boldsymbol{p}, \omega, \Theta) \tag{33}$$

By incorporating $\boldsymbol{y}_0, \boldsymbol{p}, \omega$, and Θ into the bundle adjustment, both trajectory and Mars rotation parameters can be estimated in the photogrammetric processing along with the object point coordinates $\boldsymbol{x}$.

B. Combination of Radio Tracking and Image Data

After replacing the exterior position vector by physical trajectory parameters, a combined least-squares problem involving radio tracking data and three-line scanner images may be formulated, to exploit the information content of both data types in a consistent way. To avoid the difficulties arising from a rigorous combination of orbit determination and photogrammetry, a simplified, but statistically equivalent, approach is chosen. To this end, the epoch state vector and its covariance matrix, as derived from the processing of tracking measurements in a conventional orbit determination, are utilized as a priori information in the subsequent photogrammetric point determination. In this way, the operational separation of orbit determination and bundle adjustment may be preserved without sacrificing a consistent processing of both data types.

In the sequel, the basic principle of incorporating orbit determination results into the photogrammetric bundle adjustment is discussed for the estimation of the epoch state vector and the object coordinates from a vector z of radio tracking measurements and a vector $\boldsymbol{U}$ of image coordinates. For the sake of simplicity, the modeled tracking data f_z and image coordinates f_U are assumed to depend only on the vector $\boldsymbol{X}$ of object coordinates and the epoch state vector $\boldsymbol{y}_0$.

In case of a direct combination of both data types, the residuals

$$v_z = f_z(y_0) - z, \qquad v_U = f_U(X, y_0) - U \tag{34}$$

are minimized in a least-squares sense using appropriate weights $\boldsymbol{W}_{z,z}$ and $\boldsymbol{W}_{U,U}$ for tracking measurements and image coordinates. Expanding f_z and f_U around reference values $\boldsymbol{X}^0$ and $\boldsymbol{y}_0^0$ yields the linearized equations

$$v_z = \boldsymbol{A}_{z,y}\Delta \boldsymbol{y}_0 - \boldsymbol{l}_z, \qquad v_U = \boldsymbol{A}_{U,X}\Delta\boldsymbol{X} + \boldsymbol{A}_{U,y}\Delta\boldsymbol{y}_0 - \boldsymbol{l}_U \tag{35}$$

for the residuals in terms of corrections $\Delta\boldsymbol{X} = \boldsymbol{X} - \boldsymbol{X}^0$ and $\Delta\boldsymbol{y}_0 = \boldsymbol{y}_0 - \boldsymbol{y}_0^0$ to the reference values. Here

$$\boldsymbol{l}_U = \boldsymbol{U} - f_U\left(\boldsymbol{X}^0, \boldsymbol{y}_0^0\right) \tag{36}$$

Table 1 Combined set of estimation parameters

Parameter	Dimension	Description
X	10^5	Object coordinate vector of conjugate and ground control points
$(x, y, z)_i$		Body-fixed object coordinates of ith point
y_0	6	Epoch state vector
(x_0, y_0, z_0)		Inertial spacecraft position vector at reference epoch
$(\dot{x}_0, \dot{y}_0, \dot{z}_0)$		Inertial spacecraft velocity vector at reference epoch
p	10	Force model parameter vector
C_D		Drag coefficient
C_R		Solar radiation pressure coefficient
GM		Aerocentric gravitational coefficient
C_{nm}, S_{nm}		Gravity potential coefficients of degree n and order m
$(\Delta v_r, \Delta v_t, \Delta v_n)_i$		Instantaneous velocity increments due to spacecraft thrusts
q	5	Measurement model parameter vector
$\Delta\rho_i$		Range bias for ith tracking interval
$\Delta\dot{\rho}_i$		Range rate bias for ith tracking interval
ω	5	Mars rotation parameters
α_0		Right ascension of the north pole at J2000
δ_0		Declination of the north pole at J2000
$\langle\dot{\psi}\rangle$		Precession rate
W_0		Orientation of the prime meridian at J2000
$\dot{W}$		Mars rotation rate
Θ	10^3	Exterior orientation attitude vector
$(\zeta, \eta, \theta)_i$		Eulerian rotation angles at orientation image i

and

$$l_z = z - f_z(y_0^0) \tag{37}$$

denote the difference between observed and the modeled reference measurements, whereas the Jacobians $A_{U,X}$, $A_{U,y}$, and $A_{z,y}$ denote the partial derivatives of f_U and f_z with respect to the independent parameters X and y_0. The minimum of the weighted residual norm can then be obtained by solving the combined normal equations

$$\begin{pmatrix} A_{U,X}^T W_{U,U} A_{U,X} & A_{U,X}^T W_{U,U} A_{U,y} \\ A_{U,y}^T W_{U,U} A_{U,X} & A_{U,y}^T W_{U,U} A_{U,y} + A_{z,y}^T W_{z,z} A_{z,y} \end{pmatrix} \begin{pmatrix} \Delta X \\ \Delta y_0 \end{pmatrix} = \begin{pmatrix} A_{U,X}^T W_{U,U} l_U \\ A_{U,y}^T W_{U,U} l_U + A_{z,y}^T W_{z,z} l_z \end{pmatrix} \tag{38}$$

for the unknown corrections ΔX and Δy_0.

Noting that a least-squares orbit determination based on radio tracking data alone yields the estimated epoch state vector

$$y_0^a = y_0^0 + \left(A_{z,y}^T W_{z,z} A_{z,y}\right)^{-1} A_{z,y}^T W_{z,z} l_z \tag{39}$$

and the associated variance-covariance matrix

$$C_{y,y}^a = \left(A_{z,y}^T W_{z,z} A_{z,y}\right)^{-1} \tag{40}$$

the combined normal equations may also be written in a form involving neither tracking residuals nor partial derivatives of tracking data with respect to the estimated epoch state vector. Denoting by

$$l_y = y_0^a - y_0^0 \tag{41}$$

the difference between the orbit determination result and the reference state vector and by

$$W_{y,y}^a = \left(C_{y,y}^a\right)^{-1} \tag{42}$$

the inverse of the orbit determination covariance matrix, one obtains the equivalent system

$$\begin{pmatrix} A_{U,X}^T W_{U,U} A_{U,X} & A_{U,X}^T W_{U,U} A_{U,y} \\ A_{U,y}^T W_{U,U} A_{U,X} & A_{U,y}^T W_{U,U} A_{U,y} + W_{y,y}^a \end{pmatrix} \begin{pmatrix} \Delta X \\ \Delta y_0 \end{pmatrix} = \begin{pmatrix} A_{U,X}^T W_{U,U} l_U \\ A_{U,y}^T W_{U,U} l_U + W_{y,y}^a l_y \end{pmatrix} \tag{43}$$

for the estimation of the epoch state vector and the object coordinates from image data with a priori information resulting from the orbit determination.

Based on these considerations, the use of an a priori epoch state vector in the photogrammetric image processing is equivalent to a combined estimation from radio tracking data and image data after linearization of the measurement model. Even though the result of an iterated orbit determination and a subsequent iterated bundle adjustment will, in principle, deviate from the result of the combined process in case of nonlinearities, the differences are expected to be negligible in practice, unless the orbit determination result is affected by large systematic errors.

C. Estimation Parameters

The basic principle of combining information from radio tracking and image data into a single least-squares estimation can equally well be applied for an extended set of estimation parameters. A possible set of estimation parameters for the photogrammetric processing of HRSC/WAOSS images is summarized in Table 1. Depending on the ground coverage of select images and the additional control information available in the photogrametric processing, the estimation vector may comprise force model parameters, measurement model parameters, planetary rotation parameters, and exterior attitude parameters, in addition to the epoch state vector and the object coordinates of conjugate and ground control points.

V. Summary and Conclusions

To summarize, the combined approach allows the simultaneous improvement of different types of parameters employed in the modeling of tracking data and image coordinates. This includes the epoch state vector, the force model parameters, and the measurement model parameters that are conventionally estimated within the orbit determination, as well as photogrammetric parameters such as object coordinates and orientation parameters estimated within the bundle adjustment. Because of the combination of radar tracking data, which are related to the inertial spacecraft position, and optical measurements, which are related to the planetary surface, it is furthermore possible to estimate Mars rotation parameters that affect the transformation between the inertial and the body-fixed coordinate system. The combination of both data types may, therefore, provide new information on the spin axis orientation angles and the spin rate of Mars.

Statistically, the resulting process is equivalent to a combined orbit determination and bundle adjustment from radio tracking data and three-line stereo image data. From an implementation point of view, however, a clear separation of orbit determination and bundle adjustment is maintained, which reduces the required software development effort and allows a sequential processing of radio tracking and image data by different teams and organizations.

Acknowledgment

The work of T. Ohlhof was funded by the German Space Agency (DARA) under Grant 50QM9205.

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